De-orbiting of space debris by means of a towing cable and a single thruster spaceship: whiplash and tail wagging effects

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DE-ORBITING OF SPACE DEBRIS BY MEANS OF A TOWING CABLE AND A SINGLE THRUSTER SPACESHIP: WHIPLASH AND TAIL WAGGING EFFECTS

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ABSTRACT

This papers exposes two difficulties that are likely to take place during the towing of a space debris. These effects, which could trouble de-orbitation strategies, are visible on simple simulations based on a model of coupled rigid-bodies dynamics. We name them *tail wagging* and *whiplash* effects, respectively.

Key words: De-orbiting of space debris, towing cable, open-loop design.

1. INTRODUCTION

In this paper, we study a particular type of space debris de-orbiting system. This so-called *towing* system consists of *i) a spaceship* equipped with a single thruster producing a constant force both in magnitude and (relative) orientation *ii) a towing cable* mechanically connecting the debris to the spaceship.

The paper mainly focuses on studying the effectiveness of a deceleration manoeuver (or equivalently, in relative coordinates, an acceleration manoeuver) in the orbit plane. The whole mechanical system has a large number of degrees of freedom, referring to the orientation and position of the two rigid-bodies under consideration and the shape (distributed position parameters) of the towing cable. The spaceship is assumed to be of comparable size as the debris (it is typically 3-4 times smaller and lighter). Therefore, the mutual influences of the translational and rotational motions of the two bodies can not be neglected.

In view of applications, the studied system is only open-loop, *i.e.* the towing strategy is to be determined based only on *a priori* available information. No sensors shall be employed, and no corrective manoeuver can be envisioned. This limitation stresses the need for evaluating the natural variability of the generated motions when some errors on the initial conditions are present.

For this evaluation, we develop several studies and eventually we formulate several recommendations on the design of the towing system. The conclusions are that the mechanical design should incorporate some damping, and that the thrust should be concentrated on a short period of time to minimize the variability. This recommendation stems from extensive simulations that are performed using a set of coupled rigid-bodies dynamics, which derivation is exposed. Two particular effects play key roles in this study: we name them the *tail wagging effect* and the *whiplash effect* respectively. The *tail wagging effect* is the periodic oscillation of the debris which appears as a stable limit cycle obtained after a short transient during towing. It results from the initial spin velocity of the debris. The *whiplash effect* is a sudden rotation of the spaceship which occurs right after the towing cable gets straight. The sudden straightening can occur relatively often, especially due to initial misalignments.

The paper is organized as follows. In Section 2, we sketch the context of active debris removal and expose a typical case under consideration (the ENVISAT satellite). We recall some results on its residual tumbling dynamics which makes it a difficult to catch object. Then, we detail the de-orbiting system under consideration in the article. Necessary notations to establish the dynamics of this coupled mechanical system in 2-dimension are given. In Section 3, we derive these dynamics. The Euler-Lagrange formalism is employed, using generalized coordinates to define the dynamics. The towing cable may be split into two independent parts connected via a spring-damper subsystem. The damper turns out to be useful to mitigate the natural oscillations occurring during towing. Then, in Section 4, several simulations are reported. Extensive results allow to study the effect of the system misalignment, and, importantly, the residual tumbling velocity of the debris. As appears, this defects cause some drift on the desired motion generated by open-loop towing strategies. Finally, in Section 5, we draw some conclusions and perspectives. We formulate some recommendations on the towing cable length, the damping system, and the magnitude of the thrust that should be employed to obtain, with good level of confidence, results that fall within the de-orbiting requirements.

2. BACKGROUND ON SPACE DEBRIS REMOVAL AND THE ENVISAT CASE

The ENVISAT satellite was launched in 2002. Its function was the observation of the Earth. Its acronym stands for “ENVIronment SATellite”. It was designed to provide several types of environmental measurements on the climate. The massive satellite (its initial mass is approximately 8200 kg) has lost communication with the Earth without any reason. On May, the 9th 2012, ESA (European Space Agency) has declared the end of its mission in space.

In the context of management of space debris which has attracted much attention in the last years [BGA09, Bon13, Ans10], it appears essential to remove this inert satellite from its low orbit, which is of paramount importance for space applications. Indeed, it represents a huge risk of explosion due to the likelihood of collision with other space debris. The number of space debris has already reached a situation that may threaten the future space programs (exponential divergence, call *Kessler syndrome* [KCP78]). ENVISAT is a threat worth studying as it could represent a (major) “contribution” to the increase in the number of debris in its orbit which would aggravate the situation further.

Being without any contact with its ground station, the satellite is completely incapable of manoeuvering. As a rigid-body, its current state (residual speed) is relatively uncertain which makes it even more difficult to choose a strategy for de-orbitation. Interestingly, several complementary studies [PHB+12] on space debris have been conducted earlier and have concluded that the residual rotation speed is likely to be relatively small. This is mostly due to the effects of induction braking encountered across the Earth magnetosphere which plays here a beneficial role [1].

An important issue worth discussion is determining which strategy to choose for de-orbitation. In this paper, we expose some difficulties of the towing strategy. The system under consideration is composed of the satellite, a *hunter* equipped with a thruster capable of putting the satellite in motion and a towing cable.

As mentioned above, the initial condition, mainly the residual satellite rotation (direction and value) is uncertain. In consequence, the influence of the rotation for the dispersion of traction will also be analyzed to cover a certain range of uncertainty, 1 to 8 degrees per second approximately.

[1]Nevertheless, the decay rate of this rotation speed depends on the geometry, material properties and rotation of the debris, and in our case it is relatively long (time constant being about one hundred days)

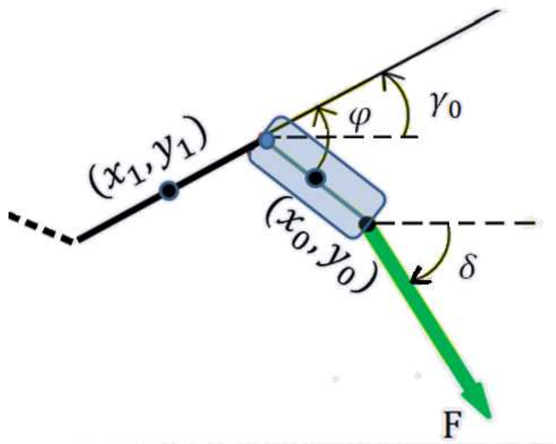

Figure 2. Close-up view of the hunter.

3. COUPLED RIGID-BODIES DYNAMICS

3.1. Systems of generalized coordinates

The debris-cable-hunter system is pictured in Figure 1.

The main idea in the proposed modeling is to consider that the towing as a collection of small rigid elements connected by their end points.

Using elementary trigonometry, it is possible to express the positions and velocities of the center of mass of every elements of the system. For simplicity of calculation, we consider the satellite as the last element of the cable and note its (planar) coordinates with respect to a fixed frame (e.g. orbit attached frame) x_n and y_n. We note its length L_N, its mass is m_N, its inertia is J_N. Respectively, the coordinates of the *hunter* will be represented by x_0 and y_0 and its size factor L_0, its mass is m_0, its inertia is J_0. For consistency, the angle of the i^{th} element of the cable with respect to the x-axis is noted γ_i, its mass is m_i, its inertia is J_i. The angle of the element nearest to the hunter is called γ_0. The angle φ designates the angle of the hunter with respect to first element of the cable. The angle δ defines the angular position of the thrust. A close-up centered on the hunter system is given in Figure 2

Without expressing the holonomic constraints, the vector $\mathbf{r}$ of non-independent coordinates can be represented as follows

$$\mathbf{r} = (x_N, y_N, x_{N-1}, y_{N-1}, \ldots, x_1, y_1, x_0, y_0, \ldots \\ \gamma_N, \gamma_{N-1}, \ldots, \gamma_1, \varphi - \gamma_0, \delta)$$

The holonomic constraints give, for the i^{th} element in the middle of the cable

$$\begin{cases} x_i = x_{i+1} + l_{i+1} \cos \gamma_{i+1} + l_i \cos \gamma_i \\ y_i = y_{i+1} + l_{i+1} \sin \gamma_{i+1} + l_i \sin \gamma_i \end{cases}$$

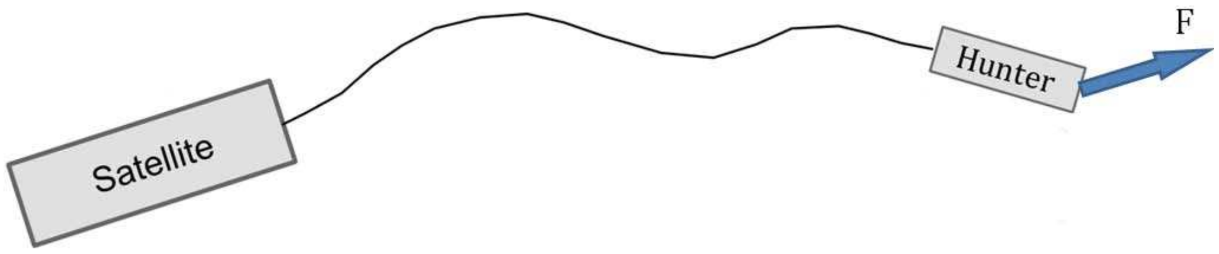

Figure 1. The debris-cable-hunter system.

$$\begin{cases} \dot{x}_i = \dot{x}_{i+1} - l_{i+1}\dot{\gamma}_{i+1}\sin\gamma_{i+1} - l_i\dot{\gamma}_i\sin\gamma_i \\ \dot{y}_i = \dot{y}_{i+1} + l_{i+1}\dot{\gamma}_{i+1}\cos\gamma_{i+1} + l_i\dot{\gamma}_i\cos\gamma_i \end{cases}$$

These expressions are valid for $N < i < 0$. The only element which coordinates and velocities are a little different is the hunter himself. An additional angle $\varphi - \gamma_0$ which is the actual angle of rotation of the axis of the hunter is considered. Subsequently, the expressions for the hunter are

$$\begin{cases} x_0 = x_1 + l_1\cos\gamma_1 + l_0\cos(\varphi - \gamma_0) \\ y_0 = y_1 + l_1\sin\gamma_1 - l_0\sin(\varphi - \gamma_0) \end{cases}$$

$$\begin{cases} \dot{x}_0 = \dot{x}_1 - l_1\dot{\gamma}_1\sin\gamma_1 - l_0(\dot{\varphi} - \dot{\gamma}_0)\sin(\varphi - \gamma_0) \\ \dot{y}_0 = \dot{y}_1 + l_1\dot{\gamma}_1\cos\gamma_1 - l_0(\dot{\varphi} - \dot{\gamma}_0)\cos(\varphi - \gamma_0) \end{cases}$$

In summary, the vector of generalized coordinates is of the form

$$\mathbf{q} = (x_N, y_N, \gamma_N, \gamma_{N-1}, \ldots, \gamma_2, \gamma_1, \gamma_0, \varphi)$$

which allows one to write the Lagrangian

$$\mathcal{L}(q, \dot{q}) = \frac{1}{2}\left(J_0(\dot{\varphi} - \dot{\gamma_0})^2 + m_0(\dot{x_0}^2 + \dot{y_0}^2)\right) + \sum_{i=1}^{N}\frac{1}{2}\left(J_i\dot{\gamma_i}^2 + m_i(\dot{x_i}^2 + \dot{y_i}^2)\right)$$

Accounting for the external non conservative forces F_i^e applied to each i^{th} of the M component of the component of $\mathbf{r}$, one obtains the Euler-Lagrange equations [LL82, Dro07]

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q_j} = \sum_{i=1}^{M} F_i^{(e)}\frac{\partial r_i}{\partial q_j}, \quad j = 1, 2, 3, \ldots, l$$

In these equations, only the forces applied to x_0 and y_0 are not zero. Nevertheless, determining the right-hand side of these equations can relatively tedious. It is convenient to use a symbolic computation software package for this task.

To improve the representativeness of the simulations we introduce some stiffness in the towing cable thanks to internal torques of the form

$$\Gamma_i = \eta(\dot{\gamma}_{i+1} - \dot{\gamma}_i)$$

where η is the stiffness parameter of the cable. The Lagrangian retains its general expression as the number of generalized coordinates remains the same. These internal torques can be interpreted as generalized similarly to F. By analogy to the force F, we find that only $\Gamma_i \neq 0$ angles are those relating to small bars, there are no components of couples for x and y. In addition, the term $\frac{\partial r_i}{\partial q_j}$ is 1 for $r_i = q_j$ and is zero for all $r_i \neq q_j$, since the angles are independent from the cartesian coordinates. Finally, the change due to the new model is the addition of a term in the Euler-Lagrange each angle γ_k, that is to say

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\gamma}_k} - \frac{\partial \mathcal{L}}{\partial \gamma_k} = \sum_{i=1}^{M} F_i^{(e)}\frac{\partial r_i}{\partial \gamma_k} + \Gamma_k, \quad k = 1, \ldots, N$$

Note that the equations governing x_n, y_n and φ do not change.

Finally, one can introduce an internal spring-damper system, following a similar approach.

At last, one shall note that the mass of the hunter is varying over time according to a constant decay rate.

4. SIMULATION RESULTS

In this section the main results achieved by extensive simulations are presented as well as some interesting phenomena, more precisely the *whiplash effect* and the *tail wagging effect.*

4.1. System specifications

For this case of study the specifications of the system elements are presented in Table 1.

Object	Mass (kg)	Length (m)	Thickness (m)
Satellite	8200	25	8
Hunter	3000∼500	3	2
Cable	10.3	50	13.5 10^{-3}

Table 1. Geometric and inertial values.

Spatial applications need some particular materials due to extreme conditions found in space (Gamma and UV radiations, heat, thermal gradient) combined to the clear objective of minimizing the mass. For the towing of such great debris as ENVISAT, the material choice that best fits the constraints appears to be the Kevlar 49, which is Kevlar covered by Aramid fiber. The *Cable* values in Table 1 can be obtained directly or by simple calculations from [DuP, Cal00] data. The considered cable, having a diameter of 13.5 mm, can handle internal forces up to 430 KN.

4.2. Impact of misalignment: *whiplash* effect

External force's orientation plays a key role in the global system dynamics. If the system is initially perfectly aligned, the cable remains stretched at all times and the multi-bars model presented in Section 3 act like the simple case of a rigid bar connecting the two bodies. However, if the external force is not aligned with the rest of the system, the cable bends, and an important phenomenon might occur: the *whiplash effect*. It consists in an aggressive straightening of the towing cable which leads to a sudden and surprising rotation of the hunter. In practice, this phenomenon could result in the failure of the cable due to frequent occurrence of very large forces at the endpoints (see Figure 3). This phenomenon must be avoided.

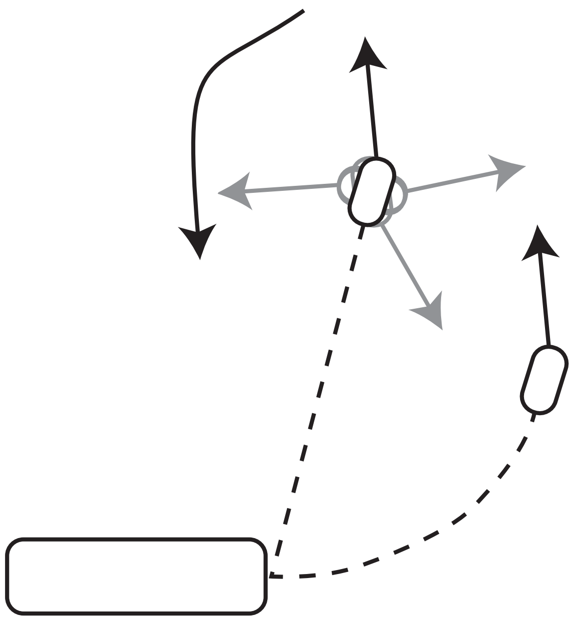

Figure 3. The whiplash effect. *When the cable gets straight after having been bend for a long time, the misalignment of the thrust causes the hunter to suddenly spin.*

4.3. Impact of debris residual tumbling: *tail wagging* effect

After this point, it is supposed that the system is always well aligned, therefore the *whiplash effect* is not expected to occur. We now detail another problem.

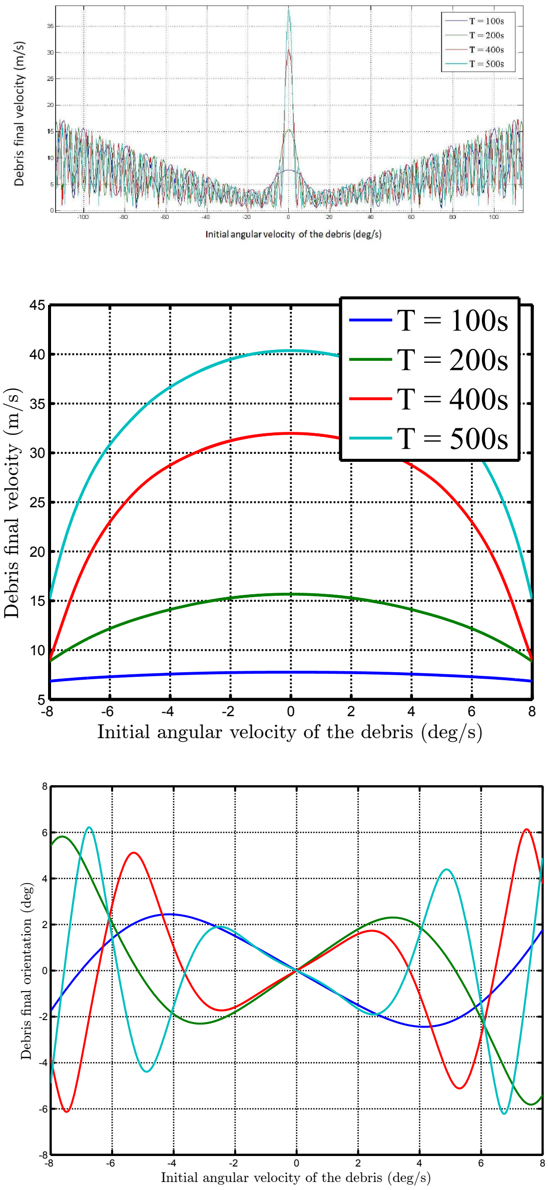

Figure 4. Debris final velocity and orientation as a function of initial residual spin.

The results reported in Figure 4 have been obtained by varying the initial angular velocity of the debris and maintaining all others initial conditions the same from one simulation to another. For example, the blue curves represent the simulations made applying an external force for 100 seconds. Between each color of curve the time of the force application is different but it is always the same within a same curve. As clearly shown in Figure 4 (bottom plot), the ΔV and orientation errors increase, with respect to the uncertainty, as the force application time increases. Thus, it is highly recommended to tow as fast as possible to minimize the variability of the system, i.e. minimize the dependence in relation to the uncertainty of the residual spin velocity.

Also, the residual tumbling of the debris implies in the second important phenomenon called *tail wagging effect*. It is the periodic oscillation of the debris which appears as a stable limit cycle obtained after a short transient during towing (Figure 5). This effect causes some drift on the desired motion generated by open-loop towing strategies as is shown in Figure 6.

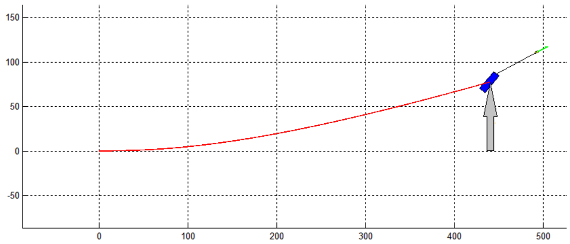

Figure 6. Drift on the desired motion.

4.4. Consequences of the spring-damper system

Finally, adding the spring-damper mechanism as described in Section 3 shows that good improvements could be achieved. Despite not being able to avoid the *tail wagging effect*, the spring and the damper can reduce its principal consequence, i.e. the spatial dispersion. Thus, using this mechanism the global system does not turn indefinitely anymore, instead it oscillates in a smooth frequency (Figure 7).

5. CONCLUSIONS AND PERSPECTIVES

In this paper, we have highlighted two malicious effects that reveal be troublesome for towing strategies applied to de-orbitation of space debris, namely the *tail wagging effect* and *whiplash effect*. From the observed simulation results several preliminary conclusions can be formulated. First, it is recommended to use a relatively high level of thrust for a short time instead of long-lasting low thrust. Of course, a particular attention should be paid to avoid failure of the cable. An internal spring-damper (e.g. located in the middle of the cable) is recommended. It will help reduce cable failure, and interestingly, it will also mitigate the secular orientation drift implied by the *tail wagging effect*. The beneficial effect of such a passive system suggests that it is possible to design a robust

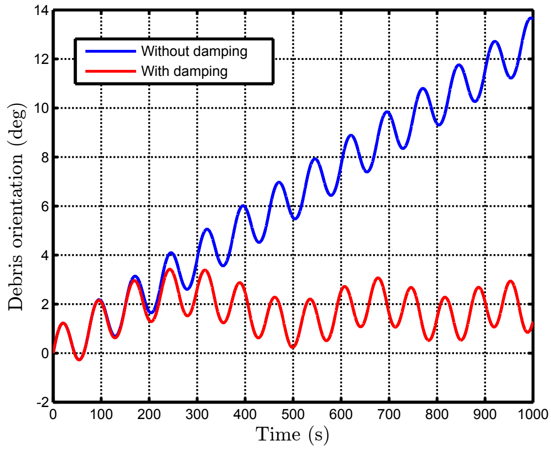

Figure 7. Mitigation of the secular orientation drift using the spring-damper mechanism.

and effective towing system employing only an open-loop strategy. Certainly, having on-board sensors will be a plus allowing one to develop closed-loop strategies to track a certain de-orbitation trajectory. This will be of importance to reject the disturbances caused by an uncertainty on the initial spin velocity of the debris, and, importantly, uncertainty of the actual point where the towing cable is attached to the debris. This raises challenging questions referring to motion planning and stabilisation of underactuated flexible mechanical systems which is a worked topic in Automatic Control [PR01, L09].

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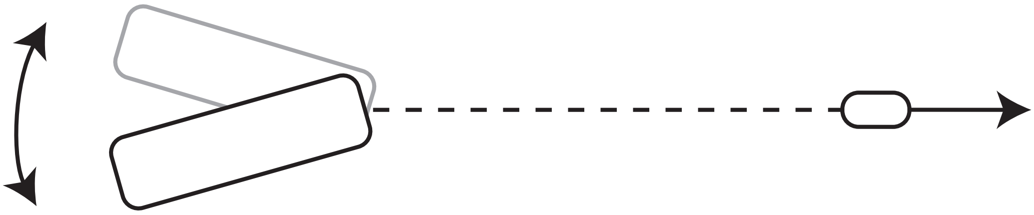

Figure 5. The tail wagging effect*. The initial spin of the debris is generating undamped oscillations as it is towed by the hunter.*

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